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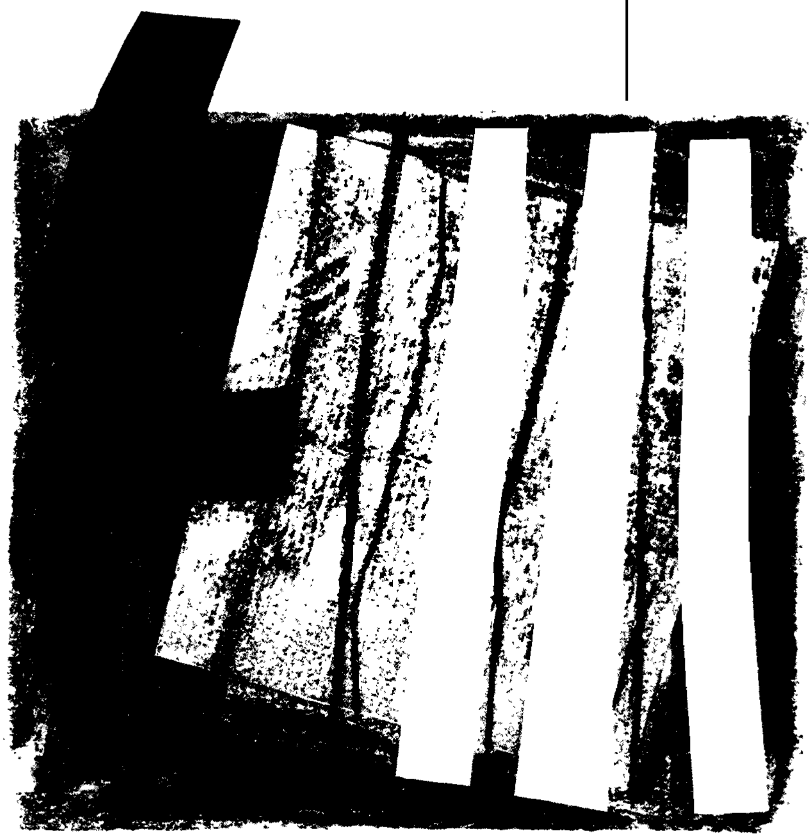

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# SPECTRAL DENSITY ESTIMATORS AT FREQUENCY ZERO FOR NONSTATIONARITY TESTS IN ARMA MODELS

Ismael Sánchez*

**Abstract**

In order to apply unit root and cointegration tests to ARMA models it is often required an estimate of the spectral density function at frequency zero. Commonly used are nonparametric estimators and the autoregressive spectral density estimator. It is well known, however, that these estimators can provoke important size and power problems, especially in presence of moving average components. This article proposes estimators based on the estimation of an ARMA model. A Monte Carlo experiment shows that the proposed procedures yields tests with better properties than competing procedures.

**Keywords:** Autoregressive moving average; GLS detrending; Spectral Density Estimators; Unit roots.

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# 1 Introduction

The statistical analysis of models with nonstationary variables has attracted considerable attention from both theoretical and applied researchers. Since the seminal work of Dickey and Fuller (1979), many tests have been developed for testing the null hypothesis of a unit root against the alternative of stationarity. Except in special cases, one often assumes that the series to be tested is driven by serially correlated innovations. Tests should, therefore, take that serial correlation into account. This constitutes a severe problem, since it is the main source of power loss and size distortion. Typically, the more common options to deal with serial correlation are: (i) to make a test based on the pivotal t-statistic of some regression, or (ii) to use some statistics based on a consistent estimate (under the null) of the spectral density function at frequency zero. It will be assumed in this paper that the serial correlation admits an ARMA representation.

Regarding the first option, the construction of tests based on the t-statistic, the analyst should decide whether to build an ARMA model or to estimate an autoregressive approximation. The autoregressive approximation is, nowadays, the most extended procedure and it is the basis of the popular Augmented Dickey-Fuller (ADF) test (Said and Dickey 1984). Although ADF is the most widely used test, it is not the most powerful. Efficient tests based on autoregressive approximations depend on the assumptions made about the initial conditions. If the initial observations are extracted from the conditional distribution (conditional case), an efficient unit root test can be obtained by applying the generalized least squares (GLS) estimator under a fixed local alternative developed by Elliott *et al.* (1996) (hereafter the $\tau_{\mathrm{GLS}}$ test). Conversely, if the first observations are extracted from the unconditional distribution under the stationary alternative (unconditional case), an efficient test can be built using the weighted symmetric estimator test (Park and Fuller 1995; Fuller 1996 p. 568) (hereafter the $\tau_W$ test). Sánchez (2000) shows why the efficiency of these tests is related to the initial conditions. Efficient tests based on t-statistics that allow for ARMA representations are discussed by Shin and Fuller (1998), based on previous work in Pantula *et al.* (1994). These tests are based on the maximum likelihood (ML) estimation (under normality) of the parameters, where the likelihood function also depends on the assumptions on the initial values. Note that this procedure can (obviously) be applied to build an autoregressive approximation.

The tests mentioned above, based on pivotal statistics, have important drawbacks. Although tests based on autoregressive approximations are very easy to implement they suffer from the well known problem of size distortion. This problem can partly be alleviated with a careful choice of the truncation lag (see for instance Ng and Perron 1995, 2000). Nevertheless, the procedures that resolve the size problem better incur power problems and vice versa. For instance, Ng and Perron (2000) propose a modified information criterion (MIC), which significantly reduces the size inflation induced by MA components by appropriately selecting the truncation lag of the autoregressive approximation. However, this procedure tends to make the test undersize and to reduce power.

On the other hand, pivotal tests based on the ML estimation of an ARMA model have, in general, fairly good properties of size and power. Their implementation is, however, complex and the specific software required to deal with the correct likelihood function is not always available to the practitioner. This is a key aspect, since the appropriate likelihood function is related to the assumptions made about the initial conditions, and an incorrect assumption on these conditions can provoke power loss. For instance, if it is assumed that the process is covariance-stationary under the alternative (unconditional case), a pivotal test based on conditional ML estimation is not efficient. Besides, the critical values of the tests also depend on the likelihood function. In the unconditional case, the asymptotic distributions of the pivotal tests also depend on the procedure used to reach the solution (i.e. finite-step estimators or iteration until convergence). Therefore, in a real situation, the analyst should be aware of the features of the software concerning ML estimation. For instance, some statistics software does not perform unconditional ML. Some others claim to do so, but actually perform some approximate procedure that could have unknown finite sample properties if applied to a unit root test. Therefore, analysts might feel worried about the reliability of pivotal tests based on ML when carried out with their habitual software.

Regarding the second choice to deal with serial correlation, the use of an statistics based on a consistent estimate of the spectral density function at frequency zero, the most frequently used test is the Phillips-Perron test (Phillips and Perron 1988). Paralleling the case of the ADF test, in spite of its popularity, it is not the most efficient. Efficient tests based on a spectral estimator are the $P_T$ tests of Elliott *et al.* (1996) and Elliott (1999). In the AR(1) case, the $P_T$ tests are, under normality, asymptotically point optimal invariant tests under a local-to-unity alternative. Elliott *et al.* (1997) built the test for the conditional case and Elliott (1999) extended it to the unconditional case. Also of interest are the $M^{\mathrm{GLS}}$ tests of Ng and Perron (2000) (see also Perron and Ng 1996). These tests are extensions of the $M$ tests of Stock (1990) (which, in turn, are modifications of the popular Phillips-Perron test) to the case where a local-to-unity alternative is considered. Ng and Perron (2000) analyze the behaviour of three different $M^{\mathrm{GLS}}$tests. Since they all have comparable performance, attention here will be restricted to the $MZ_{\alpha}^{\mathrm{GLS}}$ test.

The behaviour of these tests is highly sensitive to the estimator of the spectral density. Typically, there have been two alternative estimators in the literature: the non-parametric sum-of-covariances estimator of Phillips (1987) and Phillips and Perron (1988) and the autoregressive estimator of Stock (1990) (see also Stock 1994 and Perron and Ng 1998). It is well known that these two estimators can incur severe size distortions, especially in the presence of MA components. This fact has traditionally discouraged analysts from the use of these procedures and has made the autoregressive approximation approach the preferred choice. Ng and Perron (2000) show that the application of their MIC procedure to the autoregressive spectral estimator, together with a GLS detrending of the data, considerably improves the behaviour of these tests. However, as in the previous case, this procedure tends to make the test undersize and to reduce power.

This paper proposes spectral density estimators that allow for the estimation of ARMA models. The application of the proposed estimators yields tests with desirable size and power properties. The proposed procedures, explained in Section 2, are based on the estimation of an error correction equation using least squares (LS) or ML. Section 3 presents a limited Monte Carlo experiment showing that the performance of the tests improves significantly with the proposed estimators. Section 4 summarizes the conclusions.

## 2 The new estimators

Let $\{y_t\}$ be a discrete stochastic process. Let us assume that this process contains a deterministic component $d_t$ and a pure stochastic component $x_t$; namely, $y_t = d_t + x_t$. It will be assumed that the deterministic component consists of a mean: $d_t = \mu$; or a deterministic trend: $d_t = \mu + \delta t$. The pure stochastic part will have the following structure: $x_t = \rho x_{t-1} + u_t$, with $E(u_t) = 0$ and $E(u_t) = \sigma_u^2$. The null hypothesis of a unit root corresponds with the case $\rho = 1$ and the stationary alternative with $|\rho| < 1$. Let us assume that $u_t$ is a stationary and invertible process that admits an ARMA($p$,$q$) representation, $\phi(B)u_t = \theta(B)a_t$, where $a_t$ is a sequence of iid random variables with $E(a_t) = 0$ and $E(a_t^2) = \sigma^2$. The data-generating process is, therefore, $y_t = \mu + \delta t + x_t$; $x_t = \rho x_{t-1} + u_t$; $u_t = \psi(B)a_t$, with $\psi(B) = \phi(B)^{-1}\theta(B)$. This process can also be expressed as $y_t = \mu(1-\rho) + \delta\rho + \delta(1-\rho)t + \rho y_{t-1} + \psi(B)a_t$.

Many unit root tests require an estimate of the spectral density at frequency zero $\omega^2$, where $\omega^2 = \sigma^2\psi(1)^2 = \sigma^2\left(1 - \sum_{i=1}^{q}\theta_i\right)^2\left(1 - \sum_{j=1}^{p}\phi_j\right)^{-2}$. There are several well known estimators of $\omega^2$ in the literature. The most popular estimators are the sum-of-covariances (SC) estimator of Phillips (1987) and Phillips and Perron (1988), and the autoregressive estimate of the spectral density at frequency zero. The autoregressive estimate was first proposed by Stock (1990) and is defined as $\hat{\omega}_{AR}^2 = \hat{\sigma}_{AR}^2\left(1 - \sum_{j=1}^{k}\hat{\phi}_j^*\right)^{-2}$, where $\hat{\sigma}_{AR}^2 = (T-k)^{-1}\sum_{t=k+1}^{T}\hat{\varepsilon}_{tk}^2$, with $\hat{\phi}_j^*$ and $\hat{\varepsilon}_{tk}$ obtained from the ordinary least squares (OLS) estimation of the autoregression in error correction form

$$\Delta y_t = d_t + \alpha y_{t-1} + \sum_{j=1}^{k}\phi_j^*\Delta y_{t-j} + \varepsilon_{tk}.$$

There are several procedures in the literature for choosing the truncation lag $k$ (see, for instance, Ng and Perron, 1995, 2000). Recently, Ng and Perron (2000) proposed the estimation of $\omega^2$ using data previously detrended by a local to unity GLS. This estimator of $\omega^2$ is $\hat{\omega}_{GLS-AR}^2 = \tilde{\sigma}_{AR}^2\left(1 - \sum_{j=1}^{k}\tilde{\phi}_j^*\right)^{-2}$, where $\tilde{\sigma}_{AR}^2 = (T-k)^{-1}\sum_{t=k+1}^{T}\tilde{v}_{tk}^2$, with $\hat{\phi}_j^*$ and $\tilde{v}_{tk}$ obtained from the OLS estimation of the autoregression

$$\Delta y_t^c = \alpha y_{t-1}^c + \sum_{j=1}^{k}\phi_j^*\Delta y_{t-j}^c + v_{tk};$$

where $y_t^c$ are GLS detrended data using $\rho_c = 1 - c/T$. Ng and Perron (2000) also propose a new information criteria, MIC, to select the truncation lag $k$. In this paper, new estimators of $\omega^2$ based on the estimation of the correct model are proposed. The first estimator is defined as

$$\hat{\omega}_{ARMA}^2 = \hat{\sigma}^2\frac{\left(1 - \sum_{i=1}^{q}\hat{\theta}_i\right)^2}{\left(1 - \sum_{j=1}^{p}\hat{\phi}_j^*\right)^2}. \tag{1}$$

where $\hat{\theta}_j$, and $\hat{\phi}_i^*$ come from the estimation of the error correction regression

$$\Delta y_t = d_t + \alpha y_{t-1} + \sum_{j=1}^{p}\phi_j^*\Delta y_{t-j} + a_t + \sum_{i=1}^{q}\theta_i a_{t-i}; \tag{2}$$

and $\hat{\sigma}^2$ is the variance of the residuals. Different estimation methods are analyzed: least squares and maximum likelihood. All that is required for $\hat{\omega}^2_{ARMA}$ to be used in asymptotic unit root tests is that they converge to $\omega^2$ under the null hypothesis of a unit root and that $T\hat{\omega}^2_{ARMA}$ diverges under the stationary alternative. It can easily be checked, that $\hat{\omega}^2_{ARMA}$ is a consistent estimators under the null if the estimates $\hat{\theta}_j$ and $\hat{\phi}_i$ are root-$T$ consistent (see, for instance Shin and Lee, 2000). In addition, under the stationary alternative it converges to a constant. There are also many well known identification procedures that can help to identify the ARMA model (see, for instance Koreisha and Pukkila 1995 and references therein). Alternatively, a second estimator, $\hat{\omega}^2_{GLS-ARMA}$, based on GLS detrended data can also be defined in the same way as (1), but with $\hat{\theta}_j$, $\hat{\phi}^*_i$, and $\hat{\sigma}^2$ obtained from the regression

$$\Delta y^c_t = \alpha y^c_{t-1} + \sum_{j=1}^{p} \phi^*_j \Delta y^c_{t-j} + \varepsilon_t + \sum_{j=1}^{q} \theta_i \varepsilon_{t-j}. \tag{3}$$

This possibility of constructing an ARMA model to estimate $\omega^2$ makes a key distinction with respect to $\hat{\omega}^2_{AR}$, $\hat{\omega}^2_{GLS-AR}$, and $\hat{\omega}^2_{SC}$. This can have important practical implications, since many real time series have MA components. Besides, it is well documented in the literature that the presence of MA components can severely distort the size of unit root tests, especially if the MA polynomial contains large positive roots (see, for instance, Schwert 1989). Therefore, it is important for an estimate of $\omega^2$ to deal with MA components efficiently.

## 3 Finite sample performance.

The finite sample performance of $P_T$, $\mathrm{MZ}^{GLS}_{\alpha}$, $\tau_{GLS}$, and $\tau_W$ is analyzed when the underlying process is

$$y_t = \rho y_{t-1} + a_t - \theta a_{t-1}, \tag{4}$$

with $a_t \sim N(0,1)$ and sample size $T = 100$. Experiments were made for both the conditional case ($y_1 = a_1$) and the unconditional case. In the unconditional case, and for $\rho < 1$, a set of 50 initial observations were generated and discarded to avoid the effect of initial values.

Four different estimators of $\omega^2$ were considered. Namely, $\hat{\omega}^2_{AR}$, $\hat{\omega}^2_{GLS-AR}$, and the proposed estimators $\hat{\omega}^2_{ARMA}$ and $\hat{\omega}^2_{GLS-ARMA}$. The SC estimator $\hat{\omega}^2_{SC}$ was not included in this comparison since it is well documented in the literature that, in this context, $\hat{\omega}^2_{AR}$ is superior (Perron and Ng 1998). The estimator $\hat{\omega}^2_{AR}$

is based on a regression with data previously detrended by OLS, whereas in $\hat{\omega}^2_{GLS-AR}$ data are previously detrended by GLS under the alternative $\rho_c = 1 - c/T$, with $c = 7$ if $d_t = \mu$, and $c = 13.5$ if $d_t = \mu + \delta t$. The choice of the truncation lag $k$ in $\hat{\omega}^2_{AR}$, $\hat{\omega}^2_{GLS-AR}$, $\tau_{GLS}$, and $\tau_W$ was made with the BIC and also with the MIC proposed in Ng and Perron (2000). Whenever the BIC is used, $k$ is restricted to be $3 \leq k \leq 8$, as in Elliott *et al.* (1996). For the MIC, the restriction $0 \leq k \leq 10(T/100)^{1/4}$ is used, as in Ng and Perron (2000). The performance of the $\tau_W$ test was very poor (extremely oversize) if the lag order is selected by minimizing the MIC along the weighted symmetric autoregressions. For this reason, the lag order selection for $\tau_W$ using the MIC is based on the minimization of this criteria along ordinary error correction autoregressions, as in the ADF test.

The $P_T$ and $MZ^{GLS}_\alpha$ tests require the value of $c$ in $\rho_c = 1 - c/T$. In the non-stationary case, the values are $c = 7$ if $d_t = \mu$, and $c = 13.5$ if $d_t = \mu + \delta t$. In the stationary case, $c = 10$ for both $d_t = \mu$, and $d_t = \mu + \delta t$. These are the recommended values in Elliott *et al.* (1997) and Elliott (1999) for the $P_T$ test. Sánchez (2000) shows that these values of $c$ are also valid for the $MZ^{GLS}_\alpha$ test.

The estimators $\hat{\omega}^2_{ARMA}$ and $\hat{\omega}^2_{GLS-ARMA}$ are based on the estimation of an ARMA(1,1) model. Different estimation methods have been analyzed: least squares, exact maximum likelihood and marginal maximum likelihood. In all cases the restrictions $|\hat{\rho}| \leq 1$ and $|\hat{\theta}| \leq 1$ were used. The performance of the tests using least squares is, in general, very similar to or slightly worse than the exact maximum likelihood method. Therefore, for the sake of brevity, only results based on maximum likelihood methods are reported. Computations were made with the popular NAG routine G13bef.

Since the estimation methods are nonlinear, a set of initial values should be provided. The performance of the tests when $\theta \leq 0$ is very robust to these initial values. However, when $\theta > 0$, the tests are highly sensitive to the initial values, especially when $d_t = \mu + \delta t$. Since the region $\theta > 0$ is very frequent in real data, it can be concluded that an appropriate choice of the initial values is, then, a critical factor for a correct unit root detection. Two different types of initial values were analyzed; first, a set of random initial values; and second, a set of fixed initial values. The random initial values are obtained from a consistent estimates of the parameters. They have been obtained using the relationship between the ARMA parameters and the $\pi$-weights of the autoregressive representation. This representation holds $\pi(B)x_t = a_t$, $\pi(B) = (1-\rho B)(1-\theta B)^{-1}$, and, therefore, $\pi_1 = \rho - \theta$ and $\pi_2 = \theta(\rho - \theta)$. It is verified that $\theta = \pi_2/\pi_1$ and $\rho = \pi_1 - \theta$. To apply this result, an AR(6) has been fitted by OLS to the series. Different sets of fixed initial values for the parameters $\rho$ and $\theta$ $(\rho_0, \theta_0)$ have been analyzed. Only those values that have yielded better performance are reported. They are the following: (i) for $\hat{\omega}^2_{ARMA}$: $\rho_0 = 0.95$ and $\theta_0 = 0.90$; (ii) for $\hat{\omega}^2_{GLS-ARMA}$ : $\theta_0 = 0.50$ and for $\rho_0$ the best option is to use the same random value obtained with above mentioned AR(6).

Finite sample critical values were used for all the tests. They were obtained from 100,000 Monte Carlo replications from the model $y_t = y_{t-1} + a_t$, $y_1 = a_1$. Empirical size and power, based on 10,000 replications, are summarized in Tables ?? and ?? for the tests that are based on an estimation of $\omega^2$ : $P_T$ and $\mathrm{MZ}_\alpha^{GLS}$ tests. Tables ?? and ?? contain the results for the tests based on t-statistics: $\tau_{GLS}$ and $\tau_W$ tests. The same seeds are used for all the tests.

Conclusions are apparent. In general, and both for $P_T$ and $\mathrm{MZ}_\alpha^{GLS}$ tests, the best results (high power with size close to the nominal) are obtained by the proposed estimators $\hat{\omega}^2_{ARMA}$ and $\hat{\omega}^2_{GLS-ARMA}$. Tests based on BIC, both tests based on t-statistics and on $\omega^2$, can exhibit a very high size distortion, especially when $\theta$ is large. Tests that use MIC tend to be undersize at low values of $\theta$, and they are oversize for negative values of $\theta$. Besides, tests based on MIC show a severe loss of power. The estimator $\hat{\omega}^2_{ARMA}$ tends to oversize tests when $\theta = 0.8$ and random initial values are used. This effect, however, is alleviated if fixed initial values are used, especially if $d_t = \mu + \delta t$. The estimator $\hat{\omega}^2_{GLS-ARMA}$ has, in general, very good performance. It tends, however, to undersize tests for $\theta = 0.5$. Overall, the best performance is obtained by the $P_T$ test along with $\hat{\omega}^2_{ARMA}$ estimated by least squares or exact maximum likelihood and using fixed initial conditions $(\rho_0 = 0.95; \theta_0 = 0.90)$. When $d_t = \mu + \delta t$, very good results are also obtained using the $P_T$ test along with $\hat{\omega}^2_{ARMA}$ estimated by marginal likelihood with fixed initial conditions.

[Tables ?? to ?? about here]

# 4 Conclusions

This paper introduces new estimators of $\omega^2$ to apply unit root tests to an ARMA process. The estimators are obtained using an error correction regression based on the ARMA model using the usual estimation methods: least squares or maximum likelihood. A Monte Carlo experiment for the ARMA(1,1) model has shown that the use of the proposed estimators makes unit root tests very robust to the presence of MA components. The best empirical results are obtained when the estimation is made using initial values close to the unit circle, both for the autoregressive and moving average parameters. The proposed estimators are, therefore, an interesting alternative to tests based on autoregressive approximations, such as the popular ADF tests. Some directions for future research can be suggested. For instance, future research should determine the best identification procedure to build the ARMA model. It could be possible that, in some situations, the best model to estimate $\omega^2$ is not the correct model. The results of this article also encourage further applied research, since the proposed estimators can change the results of previous analysis based on inefficient testing procedures.

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Table 1: Empirical size and power of $P_T$ and $MZ_\alpha^{GLS}$ for 5% level using alternative spectral density estimators. T=100. 10,000 replic. Model: $(1-\rho B)(y_t-d_t)=(1-\theta B)a_t$. UNCONDITIONAL CASE.

| Estim. Method | Estimator | Initial values | Test | $\rho$ | $d_t=\mu$ | | | | | $d_t=\mu+\delta t$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $\theta$ | | | | | $\theta$ | | | | |
| | | | | | -0.8 | -0.5 | 0 | 0.5 | 0.8 | -0.8 | -0.5 | 0 | 0.5 | 0.8 |
| Exact Likelihood | $\hat{\omega}^2_{ARMA}$ | AR(6) | $P_T$ | 1.00 | 0.055 | 0.056 | 0.056 | 0.037 | 0.114 | 0.061 | 0.066 | 0.067 | 0.049 | 0.272 |
| | | | | 0.90 | 0.563 | 0.563 | 0.521 | 0.288 | 0.537 | 0.301 | 0.304 | 0.282 | 0.172 | 0.553 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.072 | 0.073 | 0.072 | 0.045 | 0.118 | 0.099 | 0.101 | 0.101 | 0.065 | 0.282 |
| | | | | 0.90 | 0.623 | 0.619 | 0.574 | 0.315 | 0.543 | 0.409 | 0.412 | 0.376 | 0.209 | 0.565 |
| | | $\rho_0$=0.95, $\theta_0$=0.90 | $P_T$ | 1.00 | 0.055 | 0.056 | 0.056 | 0.037 | 0.051 | 0.061 | 0.066 | 0.067 | 0.048 | 0.115 |
| | | | | 0.90 | 0.564 | 0.563 | 0.521 | 0.287 | 0.232 | 0.301 | 0.304 | 0.282 | 0.168 | 0.225 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.072 | 0.073 | 0.072 | 0.044 | 0.055 | 0.099 | 0.101 | 0.101 | 0.064 | 0.125 |
| | | | | 0.90 | 0.623 | 0.619 | 0.574 | 0.314 | 0.238 | 0.409 | 0.412 | 0.376 | 0.206 | 0.237 |
| | $\hat{\omega}^2_{GLS-ARMA}$ | AR(6) | $P_T$ | 1.00 | 0.054 | 0.053 | 0.047 | 0.024 | 0.049 | 0.054 | 0.062 | 0.055 | 0.022 | 0.082 |
| | | | | 0.90 | 0.550 | 0.543 | 0.457 | 0.172 | 0.251 | 0.291 | 0.286 | 0.233 | 0.079 | 0.202 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.069 | 0.069 | 0.059 | 0.025 | 0.049 | 0.093 | 0.094 | 0.079 | 0.025 | 0.083 |
| | | | | 0.90 | 0.605 | 0.597 | 0.502 | 0.181 | 0.251 | 0.393 | 0.386 | 0.313 | 0.181 | 0.204 |
| | | $\rho_0$=AR(6), $\theta_0$=0.50 | $P_T$ | 1.00 | 0.052 | 0.052 | 0.046 | 0.024 | 0.048 | 0.059 | 0.062 | 0.055 | 0.022 | 0.049 |
| | | | | 0.90 | 0.542 | 0.537 | 0.452 | 0.175 | 0.235 | 0.291 | 0.286 | 0.233 | 0.077 | 0.134 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.067 | 0.068 | 0.058 | 0.025 | 0.056 | 0.093 | 0.094 | 0.079 | 0.029 | 0.051 |
| | | | | 0.90 | 0.594 | 0.587 | 0.496 | 0.186 | 0.263 | 0.393 | 0.386 | 0.313 | 0.092 | 0.137 |
| Marginal Likelihood | $\hat{\omega}^2_{ARMA}$ | AR(6) | $P_T$ | 1.00 | 0.055 | 0.055 | 0.052 | 0.031 | 0.099 | 0.061 | 0.064 | 0.058 | 0.031 | 0.234 |
| | | | | 0.90 | 0.562 | 0.555 | 0.450 | 0.243 | 0.503 | 0.299 | 0.295 | 0.251 | 0.116 | 0.513 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.071 | 0.072 | 0.067 | 0.035 | 0.101 | 0.098 | 0.097 | 0.087 | 0.043 | 0.239 |
| | | | | 0.90 | 0.621 | 0.611 | 0.550 | 0.261 | 0.506 | 0.407 | 0.398 | 0.335 | 0.142 | 0.523 |
| | | $\rho_0$=0.95, $\theta_0$=0.90 | $P_T$ | 1.00 | 0.055 | 0.055 | 0.052 | 0.031 | 0.034 | 0.061 | 0.064 | 0.058 | 0.030 | 0.068 |
| | | | | 0.90 | 0.562 | 0.555 | 0.500 | 0.241 | 0.182 | 0.299 | 0.294 | 0.250 | 0.110 | 0.164 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.071 | 0.072 | 0.067 | 0.035 | 0.036 | 0.098 | 0.097 | 0.087 | 0.041 | 0.073 |
| | | | | 0.90 | 0.621 | 0.611 | 0.550 | 0.259 | 0.186 | 0.407 | 0.398 | 0.335 | 0.137 | 0.171 |
| | $\hat{\omega}^2_{GLS-ARMA}$ | AR(6) | $P_T$ | 1.00 | 0.055 | 0.057 | 0.048 | 0.023 | 0.049 | 0.066 | 0.065 | 0.053 | 0.021 | 0.080 |
| | | | | 0.90 | 0.557 | 0.549 | 0.461 | 0.172 | 0.251 | 0.292 | 0.288 | 0.230 | 0.071 | 0.200 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.069 | 0.069 | 0.059 | 0.025 | 0.049 | 0.093 | 0.094 | 0.079 | 0.029 | 0.083 |
| | | | | 0.90 | 0.605 | 0.597 | 0.502 | 0.181 | 0.251 | 0.393 | 0.386 | 0.313 | 0.094 | 0.204 |
| | | $\rho_0$=AR(6), $\theta_0$=0.50 | $P_T$ | 1.00 | 0.054 | 0.056 | 0.048 | 0.023 | 0.047 | 0.066 | 0.065 | 0.053 | 0.020 | 0.047 |
| | | | | 0.90 | 0.549 | 0.542 | 0.457 | 0.175 | 0.226 | 0.292 | 0.288 | 0.230 | 0.070 | 0.131 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.067 | 0.068 | 0.058 | 0.025 | 0.056 | 0.093 | 0.094 | 0.079 | 0.029 | 0.051 |
| | | | | 0.90 | 0.594 | 0.587 | 0.496 | 0.186 | 0.263 | 0.393 | 0.386 | 0.313 | 0.092 | 0.137 |
| AR(MIC) | $\hat{\omega}^2_{AR}$ | | $P_T$ | 1.00 | 0.082 | 0.054 | 0.035 | 0.038 | 0.070 | 0.101 | 0.057 | 0.029 | 0.043 | 0.145 |
| | | | | 0.90 | 0.289 | 0.278 | 0.258 | 0.250 | 0.369 | 0.156 | 0.134 | 0.100 | 0.157 | 0.382 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.097 | 0.066 | 0.042 | 0.045 | 0.074 | 0.142 | 0.086 | 0.047 | 0.060 | 0.165 |
| | | | | 0.90 | 0.317 | 0.309 | 0.294 | 0.270 | 0.380 | 0.214 | 0.196 | 0.159 | 0.205 | 0.407 |
| | $\hat{\omega}^2_{GLS-AR}$ | | $P_T$ | 1.00 | 0.085 | 0.050 | 0.033 | 0.041 | 0.056 | 0.088 | 0.051 | 0.028 | 0.027 | 0.059 |
| | | | | 0.90 | 0.295 | 0.273 | 0.270 | 0.235 | 0.296 | 0.137 | 0.107 | 0.077 | 0.097 | 0.205 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.096 | 0.060 | 0.042 | 0.045 | 0.058 | 0.115 | 0.073 | 0.043 | 0.045 | 0.077 |
| | | | | 0.90 | 0.317 | 0.303 | 0.300 | 0.251 | 0.311 | 0.193 | 0.160 | 0.128 | 0.149 | 0.233 |
| AR(BIC) | $\hat{\omega}^2_{AR}$ | | $P_T$ | 1.00 | 0.140 | 0.109 | 0.086 | 0.086 | 0.384 | 0.283 | 0.188 | 0.141 | 0.124 | 0.481 |
| | | | | 0.90 | 0.582 | 0.564 | 0.522 | 0.540 | 0.920 | 0.534 | 0.451 | 0.387 | 0.367 | 0.814 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.161 | 0.126 | 0.101 | 0.100 | 0.412 | 0.339 | 0.227 | 0.181 | 0.160 | 0.548 |
| | | | | 0.90 | 0.612 | 0.603 | 0.559 | 0.574 | 0.933 | 0.590 | 0.564 | 0.464 | 0.438 | 0.860 |
| | $\hat{\omega}^2_{GLS-AR}$ | | $P_T$ | 1.00 | 0.142 | 0.106 | 0.086 | 0.086 | 0.357 | 0.255 | 0.166 | 0.127 | 0.095 | 0.375 |
| | | | | 0.90 | 0.580 | 0.556 | 0.536 | 0.528 | 0.894 | 0.479 | 0.414 | 0.340 | 0.294 | 0.711 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.158 | 0.127 | 0.099 | 0.099 | 0.391 | 0.293 | 0.207 | 0.157 | 0.131 | 0.455 |
| | | | | 0.90 | 0.609 | 0.597 | 0.574 | 0.561 | 0.924 | 0.542 | 0.487 | 0.421 | 0.382 | 0.785 |

Table 2: Empirical size and power of $P_T$ and $MZ_\alpha^{GLS}$ for 5% level using alternative spectral density estimators. T=100. 10,000 replic. Model: $(1-\rho B)(y_t - d_t) = (1-\theta B)a_t$. CONDITIONAL CASE.

| | | | | | $d_t=\mu$ | | | | | $d_t=\mu+\delta t$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Estim. Method | Estimator | Initial values | Test | $\rho$ | $\theta$ | | | | | $\theta$ | | | | |
| | | | | | -0.8 | -0.5 | 0 | 0.5 | 0.8 | -0.8 | -0.5 | 0 | 0.5 | 0.8 |
| Exact Likelihood | $\hat{\omega}^2_{ARMA}$ | AR(6) | $P_T$ | 1.00 | 0.055 | 0.056 | 0.057 | 0.044 | 0.113 | 0.062 | 0.066 | 0.069 | 0.049 | 0.271 |
| | | | | 0.90 | 0.747 | 0.733 | 0.649 | 0.348 | 0.527 | 0.354 | 0.357 | 0.321 | 0.186 | 0.549 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.060 | 0.063 | 0.062 | 0.045 | 0.115 | 0.089 | 0.092 | 0.092 | 0.063 | 0.279 |
| | | | | 0.90 | 0.758 | 0.741 | 0.670 | 0.355 | 0.535 | 0.438 | 0.436 | 0.395 | 0.219 | 0.564 |
| | | $\rho_0$=0.95, $\theta_0$=0.90 | $P_T$ | 1.00 | 0.055 | 0.056 | 0.057 | 0.043 | 0.050 | 0.062 | 0.066 | 0.069 | 0.048 | 0.115 |
| | | | | 0.90 | 0.747 | 0.732 | 0.649 | 0.347 | 0.231 | 0.354 | 0.357 | 0.321 | 0.182 | 0.234 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.060 | 0.063 | 0.062 | 0.045 | 0.053 | 0.089 | 0.092 | 0.092 | 0.062 | 0.123 |
| | | | | 0.90 | 0.758 | 0.741 | 0.670 | 0.353 | 0.236 | 0.438 | 0.436 | 0.395 | 0.217 | 0.249 |
| | $\hat{\omega}^2_{GLS-ARMA}$ | AR(6) | $P_T$ | 1.00 | 0.053 | 0.054 | 0.050 | 0.030 | 0.052 | 0.059 | 0.060 | 0.057 | 0.025 | 0.083 |
| | | | | 0.90 | 0.731 | 0.717 | 0.585 | 0.235 | 0.268 | 0.340 | 0.335 | 0.275 | 0.097 | 0.202 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.057 | 0.059 | 0.054 | 0.030 | 0.052 | 0.084 | 0.086 | 0.076 | 0.031 | 0.084 |
| | | | | 0.90 | 0.739 | 0.725 | 0.605 | 0.234 | 0.268 | 0.423 | 0.415 | 0.343 | 0.115 | 0.204 |
| | | $\rho_0$=AR(6), $\theta_0$=0.50 | $P_T$ | 1.00 | 0.051 | 0.053 | 0.049 | 0.031 | 0.066 | 0.059 | 0.060 | 0.057 | 0.025 | 0.051 |
| | | | | 0.90 | 0.711 | 0.699 | 0.570 | 0.271 | 0.242 | 0.340 | 0.335 | 0.275 | 0.096 | 0.136 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.056 | 0.058 | 0.053 | 0.031 | 0.064 | 0.084 | 0.086 | 0.076 | 0.030 | 0.052 |
| | | | | 0.90 | 0.723 | 0.710 | 0.594 | 0.258 | 0.248 | 0.422 | 0.415 | 0.343 | 0.113 | 0.138 |
| Marginal Likelihood | $\hat{\omega}^2_{ARMA}$ | AR(6) | $P_T$ | 1.00 | 0.055 | 0.056 | 0.054 | 0.036 | 0.099 | 0.062 | 0.063 | 0.059 | 0.032 | 0.233 |
| | | | | 0.90 | 0.746 | 0.727 | 0.631 | 0.305 | 0.498 | 0.351 | 0.344 | 0.287 | 0.130 | 0.504 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.060 | 0.061 | 0.060 | 0.036 | 0.100 | 0.088 | 0.089 | 0.083 | 0.041 | 0.238 |
| | | | | 0.90 | 0.756 | 0.735 | 0.652 | 0.306 | 0.504 | 0.437 | 0.427 | 0.359 | 0.154 | 0.515 |
| | | $\rho_0$=0.95, $\theta_0$=0.90 | $P_T$ | 1.00 | 0.055 | 0.056 | 0.054 | 0.036 | 0.034 | 0.062 | 0.063 | 0.059 | 0.031 | 0.068 |
| | | | | 0.90 | 0.746 | 0.727 | 0.631 | 0.302 | 0.184 | 0.351 | 0.344 | 0.287 | 0.126 | 0.165 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.060 | 0.061 | 0.059 | 0.036 | 0.035 | 0.088 | 0.090 | 0.082 | 0.039 | 0.072 |
| | | | | 0.90 | 0.756 | 0.735 | 0.652 | 0.302 | 0.188 | 0.437 | 0.428 | 0.359 | 0.150 | 0.174 |
| | $\hat{\omega}^2_{GLS-ARMA}$ | AR(6) | $P_T$ | 1.00 | 0.053 | 0.054 | 0.050 | 0.030 | 0.052 | 0.059 | 0.060 | 0.057 | 0.025 | 0.083 |
| | | | | 0.90 | 0.731 | 0.717 | 0.585 | 0.235 | 0.268 | 0.340 | 0.335 | 0.275 | 0.097 | 0.202 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.057 | 0.059 | 0.054 | 0.030 | 0.051 | 0.084 | 0.086 | 0.076 | 0.031 | 0.084 |
| | | | | 0.90 | 0.739 | 0.725 | 0.605 | 0.234 | 0.268 | 0.423 | 0.415 | 0.343 | 0.115 | 0.204 |
| | | $\rho_0$=AR(6), $\theta_0$=0.50 | $P_T$ | 1.00 | 0.051 | 0.053 | 0.049 | 0.031 | 0.066 | 0.059 | 0.060 | 0.057 | 0.025 | 0.051 |
| | | | | 0.90 | 0.711 | 0.699 | 0.570 | 0.271 | 0.242 | 0.340 | 0.335 | 0.275 | 0.096 | 0.136 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.056 | 0.058 | 0.053 | 0.031 | 0.064 | 0.084 | 0.086 | 0.076 | 0.030 | 0.052 |
| | | | | 0.90 | 0.723 | 0.710 | 0.594 | 0.258 | 0.248 | 0.422 | 0.415 | 0.343 | 0.113 | 0.138 |
| AR(MIC) | $\hat{\omega}^2_{AR}$ | | $P_T$ | 1.00 | 0.076 | 0.052 | 0.038 | 0.043 | 0.072 | 0.100 | 0.055 | 0.027 | 0.043 | 0.147 |
| | | | | 0.90 | 0.446 | 0.433 | 0.408 | 0.301 | 0.367 | 0.187 | 0.163 | 0.129 | 0.165 | 0.371 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.085 | 0.059 | 0.039 | 0.044 | 0.075 | 0.137 | 0.076 | 0.042 | 0.057 | 0.167 |
| | | | | 0.90 | 0.442 | 0.427 | 0.409 | 0.309 | 0.377 | 0.235 | 0.212 | 0.174 | 0.203 | 0.402 |
| | $\hat{\omega}^2_{GLS-AR}$ | | $P_T$ | 1.00 | 0.062 | 0.052 | 0.037 | 0.035 | 0.027 | 0.082 | 0.048 | 0.025 | 0.035 | 0.060 |
| | | | | 0.90 | 0.397 | 0.390 | 0.363 | 0.212 | 0.123 | 0.158 | 0.141 | 0.111 | 0.117 | 0.143 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.065 | 0.054 | 0.037 | 0.034 | 0.026 | 0.106 | 0.068 | 0.038 | 0.043 | 0.064 |
| | | | | 0.90 | 0.385 | 0.380 | 0.356 | 0.205 | 0.118 | 0.194 | 0.181 | 0.152 | 0.142 | 0.152 |
| AR(BIC) | $\hat{\omega}^2_{AR}$ | | $P_T$ | 1.00 | 0.129 | 0.099 | 0.079 | 0.085 | 0.353 | 0.279 | 0.184 | 0.137 | 0.123 | 0.472 |
| | | | | 0.90 | 0.711 | 0.714 | 0.648 | 0.589 | 0.878 | 0.580 | 0.500 | 0.424 | 0.391 | 0.807 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.142 | 0.108 | 0.089 | 0.094 | 0.374 | 0.320 | 0.227 | 0.172 | 0.152 | 0.538 |
| | | | | 0.90 | 0.715 | 0.717 | 0.663 | 0.613 | 0.901 | 0.621 | 0.564 | 0.485 | 0.449 | 0.850 |
| | $\hat{\omega}^2_{GLS-AR}$ | | $P_T$ | 1.00 | 0.097 | 0.090 | 0.069 | 0.072 | 0.247 | 0.205 | 0.155 | 0.112 | 0.092 | 0.341 |
| | | | | 0.90 | 0.660 | 0.687 | 0.609 | 0.493 | 0.593 | 0.494 | 0.464 | 0.387 | 0.319 | 0.600 |
| | | | $MZ_\alpha^{GLS}$ | 1.00 | 0.104 | 0.096 | 0.076 | 0.074 | 0.258 | 0.245 | 0.195 | 0.140 | 0.109 | 0.386 |
| | | | | 0.90 | 0.659 | 0.695 | 0.627 | 0.507 | 0.621 | 0.544 | 0.521 | 0.455 | 0.372 | 0.668 |

Table 3: Empirical size and power of $\tau_{GLS}$ and $\tau_W$ for 5% level using alternative spectral density estimators.

T=100. 10,000 replic. Model: $(1-\rho B)(y_t - d_t) = (1-\theta B)a_t$. CONDITIONAL CASE.

| | $\rho$ | $d_t = \mu$ $\theta$ | | | | | $d_t = \mu + \delta t$ $\theta$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| AR order: MIC | | -0.8 | -0.5 | 0 | 0.5 | 0.8 | -0.8 | -0.5 | 0 | 0.5 | 0.8 |
| $\tau_{GLS}$ | 1.00 | 0.036 | 0.038 | 0.031 | 0.039 | 0.079 | 0.034 | 0.033 | 0.029 | 0.037 | 0.098 |
| | 0.90 | 0.358 | 0.279 | 0.357 | 0.279 | 0.263 | 0.143 | 0.146 | 0.147 | 0.159 | 0.221 |
| $\tau_W$ | 1.00 | 0.020 | 0.024 | 0.022 | 0.036 | 0.106 | 0.007 | 0.016 | 0.024 | 0.045 | 0.183 |
| | 0.90 | 0.175 | 0.232 | 0.299 | 0.299 | 0.522 | 0.042 | 0.066 | 0.129 | 0.173 | 0.441 |
| AR order: BIC | | | | | | | | | | | |
| $\tau_{GLS}$ | 1.00 | 0.063 | 0.062 | 0.050 | 0.075 | 0.355 | 0.079 | 0.064 | 0.049 | 0.080 | 0.506 |
| | 0.90 | 0.563 | 0.599 | 0.536 | 0.560 | 0.727 | 0.275 | 0.262 | 0.219 | 0.334 | 0.802 |
| $\tau_W$ | 1.00 | 0.066 | 0.041 | 0.051 | 0.126 | 0.695 | 0.078 | 0.038 | 0.052 | 0.142 | 0.857 |
| | 0.90 | 0.480 | 0.444 | 0.527 | 0.791 | 0.999 | 0.264 | 0.171 | 0.230 | 0.521 | 0.997 |

Table 4: Empirical size and power of $\tau_{GLS}$ and $\tau_W$ for 5% level using alternative spectral density estimators.

T=100. 10,000 replic. Model: $(1-\rho B)(y_t - d_t) = (1-\theta B)a_t$. UNCONDITIONAL CASE.

| | $\rho$ | $d_t = \mu$ $\theta$ | | | | | $d_t = \mu + \delta t$ $\theta$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| AR order: MIC | | -0.8 | -0.5 | 0 | 0.5 | 0.8 | -0.8 | -0.5 | 0 | 0.5 | 0.8 |
| $\tau_{GLS}$ | 1.00 | 0.027 | 0.024 | 0.021 | 0.032 | 0.082 | 0.030 | 0.029 | 0.028 | 0.033 | 0.109 |
| | 0.90 | 0.169 | 0.173 | 0.192 | 0.212 | 0.401 | 0.118 | 0.119 | 0.127 | 0.142 | 0.299 |
| $\tau_W$ | 1.00 | 0.020 | 0.024 | 0.022 | 0.036 | 0.106 | 0.007 | 0.016 | 0.024 | 0.045 | 0.181 |
| | 0.90 | 0.132 | 0.181 | 0.241 | 0.273 | 0.515 | 0.033 | 0.060 | 0.121 | 0.169 | 0.449 |
| AR order: BIC | | | | | | | | | | | |
| $\tau_{GLS}$ | 1.00 | 0.065 | 0.052 | 0.052 | 0.074 | 0.437 | 0.079 | 0.059 | 0.049 | 0.083 | 0.564 |
| | 0.90 | 0.348 | 0.331 | 0.313 | 0.442 | 0.965 | 0.245 | 0.220 | 0.189 | 0.303 | 0.896 |
| $\tau_W$ | 1.00 | 0.067 | 0.037 | 0.051 | 0.122 | 0.690 | 0.076 | 0.033 | 0.052 | 0.148 | 0.845 |
| | 0.90 | 0.410 | 0.346 | 0.456 | 0.749 | 0.998 | 0.242 | 0.148 | 0.211 | 0.495 | 0.995 |